ISSN NO: 2230-5807

FUZZY ALMOST ii-NORMAL SPACES

Dr.S.Kamali
Assistant professor of Mathematics,
University College of Engineering, Thirukkuvalai 610204.

Abstract. The aim of this paper is to introduced and study a new class of fuzzy almost normal spaces, called fuzzy almost ii-normal spaces by using fuzzy ii-open sets and explore several properties of such a fuzzy topological space.
Key words: fuzzy closed sets, fuzzy open set, fuzzy ii-open, fuzzy ii-closed sets, fuzzy M-ii-closed, fuzzy M-ii-open, fuzzy almost ii-irresolute functions, fuzzy almost ii-normal spaces.

I.PRELIMINARIES:

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family {Aα: ∈αΛ} of fuzzy sets of X is defined by to be the mapping sup Aα (resp. inf Aα) . A fuzzy set A of X is contained in a fuzzy set B of X if A(x) ≤ B(x) for each x∈X. A fuzzy point xβ in X is a fuzzy set defined by xβ (y) = β for y=x and x(y) =0 for y ≠ x, β∈[0,1] and y ∈ X .A fuzzy point xβ is said to be quasi-coincident with the fuzzy set A denoted by xβqA if and only if β + A(x) > 1. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by AqB if and only if there exists a point x∈X such that A(x) + B(x) > 1 .A ≤ B if and only if ⌉(AqBc).A family τ of fuzzy sets of X is called a fuzzy topology on X if 0,1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy open sets and their complement are fuzzy closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed sets of A and the interior of A (denoted by int(A))is the union of all fuzzy open subsets of A.

Throughout this paper (X ,τ),(Y, σ) represent non-empty fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned .For a subset A of a fuzzy space (X,τ), cl(A) and int(A) denote the fuzzy closure and the fuzzy interior of A respectively.

Definition 1.1:A fuzzy Subset A of fuzzy topological space (X, τ) is called

1. fuzzy semi-open set if A⊆cl(int(A)) and a fuzzy semi-closed set if int(cl(A))⊆A.
2. fuzzy semi-pre open set if A⊆cl(int(cl(A)) and a fuzzy semi-pre closed set if int(cl(int(A)))⊆A
3. fuzzy regular -open set if int(cl(A))=A and a fuzzy regular -closed.

Definition 1.2:A fuzzy Subset A of fuzzy topological space (X ,τ) is called

1. fuzzy generalized closed set (briefly fuzzy g-closed) if cl(A)⊆U whenever A ⊆U and U is fuzzy open in (X ,τ)
2. fuzzy g*-closed set if cl(A)⊆U whenever A⊆U and U is fuzzy g open in (X ,τ)
3. fuzzy g**-closed set if cl(A)⊆U whenever A⊆U and U is fuzzy g* open in (X ,τ)

Definition.1.3:. A subset A of a fuzzy topological space X is called

1. fuzzy α-closed if cl(int(cl(A))) ⊆A .
2. fuzzy gα-closed if α-cl(A) ⊂U, whenever A ⊂U, and U is fuzzy α-open in X .

Vol 12 Issue 03 2023

ISSN NO: 2230-5807

3. fuzzy rgα-closed if $\alpha\text{-cl}(A) \subset U$, whenever $A \subset U$, and U is fuzzy regularly α-open in X.
4. fuzzy ii-closed if $\text{int}(\text{cl}(A)) \cap \text{cl}(\delta\text{-int}(A)) \subseteq A$.
5. fuzzy g-ii-closed if $\text{ii-cl}(A) \subset U$, whenever $A \subset U$ and U is fuzzy ii-open in X.
6. fuzzy regularly ii-open if there is a regularly open set U such that $U \subset A \subset \text{ii-cl}(U)$.
7. fuzzy rg-ii-closed if $\text{ii-cl}(A) \subset U$, whenever $A \subset U$, and U is fuzzy regular ii-open in X.

The complement of fuzzy α-closed (resp. fuzzy gα-closed, fuzzy rgα-closed, fuzzy ii-closed, fuzzy g ii-closed, fuzzy rg ii-closed) set is said to be fuzzy α-open (resp. fuzzy gα-open, fuzzy rgα-open, fuzzy ii-open, fuzzy g ii-open, fuzzy rg ii-open) set. The complement of fuzzy regularly ii-open set is said to be fuzzy regularly ii-closed set. Definitions stated in preliminaries and above, we have the following diagram:

fuzzy closed ⇒ fuzzy α-closed ⇒ fuzzy gα-closed ⇒ fuzzy rgα-closed
⇓ ⇓ ⇓
fuzzy ii-closed ⇒ fuzzy gii-closed ⇒ fuzzy rgii-closed

However the converses of the above are not true may be seen by the following examples.

2. Fuzzy Almost ii- Normal Spaces

Definition.2.1 A fuzzy topological space X is said to be fuzzy almost - normal [6] resp. fuzzy almost ii-normal) if for every pair of disjoint fuzzy sets A and B, one of which is fuzzy closed and other is fuzzy regularly closed, there exist fuzzy disjoint open(resp. fuzzy ii-open)sets U and V of X such that $A \subset U$ and $B \subset V$.

Lemma.2.1 A subset A of a fuzzy topological space X is fuzzy rgii-open iff $F \subset \text{ii-int}(A)$ whenever F is fuzzy regularly closed and $F \subset A$.

Theorem.2.1. For a fuzzy topological space X, the following are equivalent:

(a) X is fuzzy almost ii- normal.

(b) For every fuzzy closed set A and every fuzzy regularly closed set B, there exist disjoint fuzzy g ii-open sets U and V such that $A \subset U$ and $B \subset V$.

(c) For every fuzzy closed set A and every fuzzy regularly closed set B, there exist disjoint fuzzy rg ii-open sets U and V such that $A \subset U$ and $B \subset V$.

(d) For every fuzzy closed set A and every fuzzy regularly open set B containing A there exists a fuzzy g ii-open set U of X such that $A \subset U \subset \text{ii-cl}(U) \subset B$.

(e) For every fuzzy closed set A and every fuzzy regularly open set B containing A, there exists a fuzzy rg ii-open set U of X such that $A \subset U \subset \text{ii-cl}(U) \subset B$.

(f) For every pair of fuzzy disjoint sets A and B, one of which fuzzy closed and other is fuzzy regularly closed, there exist fuzzy ii-open sets U and V such that $A \subset U$ and $B \subset V$ and $U \cap V = \phi$.

Proof. (a) ⇒(b), (b) ⇒(c), (d) ⇒(e), (c) ⇒(d) , (e) ⇒(f) and (f) ⇒(a).

(a) ⇒(b). Let X be a fuzzy almost ii-normal. Let A be a fuzzy closed and B be a fuzzy regularly closed sets in X. By assumption, there exist fuzzy disjoint ii-open sets U and V such that $A \subset U$ and $B \subset V$. Since every fuzzy ii-open set is fuzzy gii-open set , U, V are fuzzy g ii- open sets such that $A \subset U$ and $B \subset V$.

(b) ⇒(c). Let A be a fuzzy closed and B be a fuzzy regularly closed sets in X. By assumption, there exist fuzzy disjoint gii-open sets U and V such that $A \subset U$ and $B \subset V$. Since every fuzzy gii-open set is fuzzy rg ii open set, U, V are fuzzy rg ii-open sets such that $A \subset U$ and $B \subset V$.

(d) ⇒(e). Let A be any fuzzy closed set and B be any fuzzy regularly open set containing A. By assumption, there exists a fuzzy g ii-open set U of X such that $A \subset U \subset$ ii-cl(U) $\subset B$. Since every fuzzy gii-open set is fuzzy rg ii-open set , there exists a fuzzy rg ii- open set U of X such that $A \subset U \subset$ ii-cl(U) $\subset B$.

(c) $\Rightarrow$(d). Let A be any fuzzy closed set and B be a fuzzy regularly open set containing A. By assumption, there exist fuzzy disjoint rg ii-open sets U and W such that $A \subset U$ and $X - B \subset W$.

By Lemma 2.1, we get,

$X - B \subset$ii-int(W) and ii-cl(U) $\cap$ii-int(W) = ϕ. Hence, $A \subset U \subset$iicl(U) $\subset X -$ii-int(W) $\subset B$.

(e) $\Rightarrow$(f). For any fuzzy closed set A and any fuzzy regularly open set B containing A . Then $A \subset X - B$ and $X - B$ is a fuzzy regularly closed. By assumption, there exists a fuzzy rg ii-open set G of X such that $A \subset G \subset$ ii-cl(G) $\subset X - B$. Put U = ii-int(G),V = X –ii-cl(G).Then U and V are fuzzy disjoint ii open sets of X such that $A \subset U$ and $B \subset V$.

(f) $\Rightarrow$(a) is obvious.

Definition.2.2.: A fuzzy function $f : X \to Y$ is called fuzzy rc-continuous [3] if for each fuzzy regular closed set F in Y, $f^{-1}(F)$ is fuzzy regularly closed in X .

Definition. 2.3.;A fuzzy function $f : X \to Y$ is called fuzzy M -ii-open (resp. fuzzy M -ii-closed) if (U)$\in$iiO(Y) (resp. f(U) $\in$iiC(Y)) for each U $\in$iiO(X) (resp. fuzzy U $\in$iiC(X)).f

Definition.2.3.A fuzzy function $f : X \to Y$ is called fuzzy almost ii-irresolute if for each $x \in X$ and each fuzzy ii-neighborhood V of f (x), ii-cl(f^{-1}(V)) is a fuzzy ii-neighborhood of x.

Theorem.2.2: If $f : X \to Y$ is fuzzy continuous M-ii-open rc-continuous and fuzzy almost ii-irresolute surjection from a fuzzy almost ii-normal space X onto a fuzzy space Y, then Y is fuzzy almost ii-normal.

Proof. Let A be a fuzzy closed set and B be a fuzzy regularly open set containing A. Then by fuzzy rc-continuity of f, $f^{-1}(A)$ is a fuzzy closed set contained in the fuzzy regularly open set $f^{-1}(B)$. Since X is fuzzy almost ii-normal, there exists a fuzzy ii-open set V in X such that $f^{-1}(A) \subset V \subset$ii-cl (V) $\subset f^{-1}(B)$ by Theorem 3.4. Then, $f(f^{-1}(A)) \subset f(V) \subset$ f(ii-cl(V)) $\subset f(f^{-1}(B))$. Since f is fuzzy M-ii-open and almost ii-irresolute surjection, it follows that f(V)$\in$iiO(Y) , we obtain $A \subset f(V) \subset$ ii-cl(f (V)) $\subset B$. Then Y is fuzzy almost ii- normal.

Theorem. 2.3. If $f : X \to Y$ is fuzzy rc-continuous M -ii-closed map from an fuzzy almost ii-normal space X onto a fuzzy space Y, then Y is fuzzy almost ii- normal

Proof. Easy to verify.

REFERENCES

[1]. Bin Sahana A. S. Mapping in fuzzy topological spaces fuzzy sets and systems, 61(1994) ,209-213.

[2]. Bin Sahana A. S. on fuzzy strongly semi continuity and fuzzy pre continuity, fuzzy sets and systems 44(1991),303-308.

[3]. Chang C.L. Fuzzy topological spaces .J. Math Anal. Appl..24(1968), 182-190.

[4]. D.Andrijevic, Semi preopen sets, Mat. Vesnik 38(1986),24-32.

[5]. George J. Klir and Bo Yuan, Fuzzy sets and fuzzy logic theory and applications Prentice Hall of India New Delhi 2003.

[6]. L.A. Zadeh, Fuzzy Sets, Inform and control. 8(1965), 338-35.

[7]. Lin.Y. M and Lou. K.M., Fuzzy topology, World Scientific Publication Singapore(1997).

[8]. Livine N. Generalized closed sets in topology Rand. Circ Mat. Palermo, 19(2)(1970) ,571-599.
[9]. Livine N. Semi Open Sets and semi continuity in topological spaces Amer. Math. Mothly,70(1963),36-41.
[10]. Lowen R. A comparison of different compactness notion on fuzzy topological spaces, J. Math. Anal. Appl. 64(1978), 446-454.
[11]. Lowen R. Fuzzy topological Spaces and fuzzy compactness in fuzzy J. Math,Anal. Appl. 56(1976),621-633.
[12]. Mahashwari S. N. and Prasad R. on s-regular spaces Glasnik Mat. Ser.III 10(3),(1975),347-350.
[13]. Mahashwari S. N. and Prasad R. Some new separation axioms Ann. Soc. Sci. Bruxelles T.-89III(1975),395-402.
[14]. Mashour A. S. , M.F. Abd. Monsef. EI.,Deeb S.N. on ptre continuous and weak precontinious mappings,Proc.Math and Phys. Soc, Egypt53(1982),47-53.
[15]. Mishra M.K., Shukla M. "Fuzzy Regular Generalized Super Closed Set" International Journal of Scientific and Research Publication ISSN2250-3153. July December 2012.
[16]. Mukerjee M.N. And Sinha S.P. Almost compact fuzzy topological spaces Mat Vasnik 41(1989),89-97.
[17]. Nanda S. On fuzzy topological Spaces fuzzy sets and systems 19(2),(1986),193-197.
[18]. P.M. Pu and Y.M. Liu, Fuzzy topology I. Neighbourhood structure of a fuzzy point and moore smith convergence.J.Math. Anal. Appl. 76(1980),571-599.
[19]. Palaniappan n. and Rao K.C. Regular Generalized closed sets Kyungpook Math. J.33(2),1993,211-219.
[20]. Pu. P.M. and Lin.Y.M., Fuzzy topology II. Product Quotient spaces. J.Math. Anal. Appl. 77(1980)20-27.
[21]. Pu.P.M. and Lin.Y.M., Fuzzy topology, I.Neighbourhood structure of a Fuzzy point Moore Smith
convergence.J.Math.Anal.Appl.76(1980)571-599.
[22]. Wong C.K on fuzzy points and local properties of fuzzy topology J. Math Anal. Appl 46(1974)316-328.
[23]. Yalvac T.H.Fuzzy Sets and functions in fuzzy Spaces J. Math Anal. Appl 126 (1987),409-423.